

Neutrino Unification

P. H. Chankowski¹, A. Ioannian^{2,3}, S. Pokorski^{1,4} and J. W. F. Valle²

¹ *Univ. of Warsaw Inst. Physics Ul. Hoża 69, 00-681, Warsaw, Poland*

² *Instituto de Física Corpuscular – C.S.I.C. – Univ. of València*

Edificio Institutos de Paterna – Apartado de Correos 2085 - 46071 València, Spain

³ *On leave from Yerevan Physics Institute, Alikhanyan Br.2, Yerevan, 375036, Armenia*

⁴ *Frontier Fellow at Fermilab, P.O. Box 500, Batavia IL 60510, USA*

(February 1, 2008)

Present neutrino data are consistent with neutrino masses arising from a common seed at some “neutrino unification” scale M_X . Such a simple theoretical ansatz naturally leads to quasi-degenerate neutrinos that could lie in the electron-volt range with neutrino mass splittings induced by renormalization effects associated with supersymmetric thresholds. In such a scheme the leptonic analogue of the Cabibbo angle θ_\odot describing solar neutrino oscillations is nearly maximal. Its exact value is correlated with the smallness of $\theta_{reactor}$. These features agree both with latest data on the solar neutrino spectra and with the reactor neutrino data. The two leading mass-eigenstate neutrinos present in ν_e form a pseudo-Dirac neutrino, avoiding conflict with neutrinoless double beta decay.

The standard model (SM) and its minimal supersymmetric extension (MSSM) fail only in accounting for the solar and atmospheric neutrino data [1–3] which strongly indicate the need for neutrino conversions, as would arise from neutrino-mass-induced oscillations [4]. It is thus reasonable to investigate simple theoretically motivated ansatz that can account for these observations.

Inspired by the idea of unification of fundamental interactions [5] here we propose the idea of neutrino unification: namely that the neutrino mass and mixings observed at low energies take a very simple form at some high energy scale M_X . Thus, we add to the basic Lagrangean the dimension-five operator [6]

$$\frac{\lambda_0 \delta_{ab}}{M_X} (\phi \ell_a) (\ell_b \phi) + H. c. \quad (1)$$

where ℓ_a denote the three lepton doublets and ϕ is the standard Higgs doublet. As a working hypothesis, we assume this operator to be characterized at the scale M_X , by a single real dimension-less parameter λ_0 . The breaking of the electroweak symmetry due to a non-zero vacuum expectation value (VEV) $\langle \phi \rangle$ will generate, in addition to the known SM masses, the seesaw-type neutrino mass operator M_ν

$$M_\nu = \frac{\langle \phi \rangle^2}{M_X} \lambda_0 \quad (2)$$

For $M_X \sim M_{Planck}$ neutrino masses would be too small to account for the atmospheric neutrino anomaly. Thus we will adopt an intermediate value for the M_X scale. In the basis in which the charged lepton Yukawa couplings are diagonal (weak basis), $\lambda_0 \delta_{ij}$ gets transformed into

$$\Lambda = \lambda_0 \Omega^T \Omega \quad (3)$$

where Ω is an arbitrary unitary matrix. After the electroweak symmetry breaking we can rotate back the neutrino fields, $\nu = \Omega^{-1} \nu'$, and in this basis (the charged lepton mass matrix is still diagonal) we get

$$\mathcal{L} \supset \frac{\lambda_0 v^2}{M_X} \nu' \nu' - \frac{g}{\sqrt{2}} \bar{e}_L \bar{\sigma}^\mu W_\mu^- U \nu' + H. c. \quad (4)$$

where $U = \Omega^\dagger$ and the fields ν' are the neutrino mass eigenstates (in this paper we number the eigenvalues so that $\Delta m_{21}^2 = m_2^2 - m_1^2 \equiv \Delta m_\odot^2$ corresponds to the solar neutrino oscillations and $\Delta m_{32}^2 = m_3^2 - m_2^2 \equiv \Delta m_{atm}^2$ corresponds to the atmospheric neutrino oscillations). In general, the mixing matrix U is characterized by three mixing angles and three CP violating phases, one Dirac plus two Majorana-type phases [7–9] and can be written as the product of V_{CKM} by

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\alpha_1} & 0 \\ 0 & 0 & e^{i\alpha_2} \end{pmatrix} \quad (5)$$

where we have chosen to parametrize V_{CKM} exactly as the Cabibbo-Kobayashi-Maskawa matrix describing quarks, with the familiar three angles and one phase δ . It is clear that with our ansatz (1) we have an additional freedom of performing any arbitrary real 3×3 rotation of the neutrinos. The general form of such a restricted matrix U can be derived by applying a convenient parametrization [7] of a general unitary matrix to the matrix $M \equiv \Omega^T \Omega$ and by imposing the symmetry

condition $M = M^T$. Using the freedom of re-phasing the charged leptons, one can show that the U matrix cannot be eliminated by redefining the neutrinos. Thus, despite the mass degeneracy amongst the neutrinos, they exhibit non-trivial mixing effects [10]. In contrast to the case investigated in ref. [10] where neutrinos were massless but non-orthonormal, here it is the fact that the mass eigenvalues have different phases which prevents one from rotating away the mixing amongst the neutrinos. Nevertheless, the structure of the mixing in our case is more restricted than the general case, since only two angles and one phase remain physical. The above construction has the virtue of explicitly listing the relevant mixing parameters for exactly degenerate Majorana neutrinos. Once the neutrino mass degeneracy is lifted by quantum corrections the mixing effects require the full matrix U with, however, a non-trivial relationship amongst the three mixing angles.

Non-zero values of the Dirac phases mean CP non-conservation, while Majorana phases of $\pi/2$ indicate different CP parities of the neutrino mass eigenstates and do not imply CP violation [11]. Thus in our case CP is conserved if the phase $\delta = 0 \bmod \pi$ and $\alpha_1, \alpha_2 = 0 \bmod \pi/2$. Our main conclusions do not depend on whether or not CP is conserved in the neutrino sector and from now on we will assume it is conserved. Different CP parities of neutrino mass eigenstates can then be accounted for by different signs of neutrino masses (which for $\alpha_1 = \pi/2$ and/or $\alpha_2 = \pi/2$ can be achieved by simply absorbing these phases in the neutrino fields).

Now we turn to the renormalization effects. One-loop renormalization group equations (RGEs) for the Λ coefficients characterizing the dimension-5 non-renormalizable terms can easily be written down both for the SM and MSSM [12]. The RGEs take a particularly useful form when written directly for mass eigenvalues and for the elements of the mixing matrix [13]. The flavour independent corrections to the $\Lambda = \mathcal{O}(1)$ coefficients are irrelevant for our discussion. The flavour-dependent corrections are due to lepton Yukawa couplings and, to a good approximation, determined by the τ Yukawa coupling. Using e.g. the results of ref. [13] one can easily verify that, with the ansatz (1), such effects cannot explain $|\Delta m_{32}^2| \gg |\Delta m_{21}^2|$ with phenomenologically acceptable mixing angles.

It is remarkable that supersymmetry can induce flavour-dependent threshold corrections associated with slepton mass splittings [14] which can dominate over the τ Yukawa corrections. We show in this paper that such corrections can lead to the desirable low energy pattern with the ansatz (1). This is possible provided the soft supersymmetry breaking scalar mass terms deviate sufficiently from universality.

The quantum corrections to our ansatz (1) in the basis defined by eq. (4) are given by [14]

$$m_\nu^{ab} = m_a \delta_{ab} + m_a (U^T I U^*)_{ab} + m_b (U^\dagger I U)_{ab} \quad (6)$$

where in our case all $|m_a| = m$ up to a common flavour-independent renormalization $\mathcal{O}(1)$ factor. Here the indices a, b refer to our starting basis used in eq. (1). The correction I (calculated in the electroweak charged lepton mass eigenstate basis) consists of two parts

$$I = I^{RG} + I^{TH} \quad (7)$$

where I^{RG} is the renormalization group correction [15] and I^{TH} are the electroweak scale threshold corrections [14]. Assuming no lepton flavour violation in other sectors of the theory (e.g. in the case of supersymmetry) the matrix I is diagonal, $I_{AB} = I_A \delta_{AB}$ (the index A refers to the electroweak basis). Without loss of generality in our discussion we take $I_\mu = 0$. We adopt the general form for the CP conserving mixing matrix U [7,16]

$$\begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13} & c_{12}c_{23} - s_{12}s_{23}s_{13} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13} & -c_{12}s_{23} - s_{12}c_{23}s_{13} & c_{23}c_{13} \end{pmatrix} \quad (8)$$

in which $\theta_{13} \equiv \theta_{reactor}$, and allow for different CP parities of neutrino eigenstates. For degenerate masses $|m_a| = m$ the matrix (8) contains one redundant angle and this degree of freedom is used to re-diagonalize the mass matrix (6). As long as we assume CP conservation we can use the standard perturbation theory for degenerate zeroth order eigenvalues in order to determine the non-trivial relation amongst the three mixing angles in eq. (8). Taking $|I_\tau| \gg |I_e|$ we easily recover the conclusions of ref. [13]. In this letter we investigate the alternative possibility that $|I_e| \gg |I_\tau|$. We begin with $m_1 = -m_2 = m_3 \equiv m$ in which case

$$m_\nu^{ab} = m \begin{pmatrix} 1 + 2U_{A1}^2 I_A & 0 & 2U_{A1}U_{A3}I_A \\ 0 & -1 - 2U_{A2}^2 I_A & 0 \\ 2U_{A1}U_{A3}I_A & 0 & 1 + 2U_{A3}^2 I_A \end{pmatrix} \quad (9)$$

Since $m_1 = m_3$, the ordinary perturbation calculus tells us that the neutrino mass basis should be chosen so that the off-diagonal 13 entry of the perturbation is zero. We therefore require that $U_{A1}U_{A3}I_A = 0$ which fixes the redundant angle in the matrix (8). With our assumptions about I_A 's ($|I_e| \gg |I_\tau|$) this gives

$$s_{13} = -\frac{s_{12}}{c_{12}} s_{23} c_{23} r + \mathcal{O}(r^2) \quad (10)$$

where $r \equiv I_\tau/I_e$. It is then trivial to compute the corrected mass eigenvalues and one finds

$$\Delta m_{atm}^2 \approx -4m^2 I_e s_{12}^2, \quad (11)$$

$$\Delta m_\odot^2 \approx -4m^2 I_e [C_{12} (1 - s_{23}^2 r) + (1 + 2c_{12}^2) s_{13}^2] \quad (12)$$

where $C_{12} \equiv c_{12}^2 - s_{12}^2$. For maximal $\theta_{12} = \pi/4$ one obtains that $\Delta m_{atm}^2 \sim I_e$ and $\Delta m_\odot^2 \sim I_e r^2$, in agreement with the experimental requirement $\Delta m_\odot^2 \ll \Delta m_{atm}^2$.

Eqs. (11) and (12) lead to the following relation between the neutrino oscillation parameters

$$\Delta m_{\odot}^2 \approx 2\Delta m_{atm}^2 [C_{12}(1 - s_{23}^2 r) + 2s_{13}^2] \quad (13)$$

which is the prediction of our model, independent of the value of I_e . Since we need $c_{12}^2 \approx s_{12}^2$ we conclude that small mixing solution to the solar neutrino problem is ruled out in our neutrino unification picture.

It is easy to check that with our assumptions about I_A 's the same mechanism works in the case $-m_1 = m_2 = m_3$. However, in the case $m_1 = m_2 = -m_3$ the condition $U_{A1}U_{A2}I_A = 0$ means that $s_{12}c_{12}c_{13}^2 = \mathcal{O}(r)$ which means that one cannot have simultaneously large solar neutrino mixing and remain consistent with the reactor experiments [17] (this is also incompatible with the SMA solution to the solar neutrino problem because for $s_{12}c_{12} \approx 0$ one gets $\Delta m_{\odot}^2 \sim \Delta m_{atm}^2$).

Since $\Delta m_{atm}^2 \approx 3 \times 10^{-3} \text{eV}^2$ is rather fixed by the analysis of the data [4] and since, as we shall see, typically $|I_e| \lesssim 10^{-3}$, it follows that $m \gtrsim \mathcal{O}(1) \text{eV}$. Thus this picture leads to neutrino masses in the range accessible to β decay and hot dark matter searches. Different CP parities of the neutrinos are however sufficient to ensure a destructive interference in the amplitude for neutrinoless double beta decay which is suppressed by the fact that the two leading mass eigenstate neutrinos in the ν_e form a pseudo-Dirac neutrino [18].

How can one realize our neutrino unification scenario? First of all, it requires the existence of new states giving a dominant contribution to I_e . Rather than extending the gauge and/or multiplet structure of the SM, we prefer to ask whether in supersymmetry one can arrange the corrections I_A to satisfy our requirements. Taking a diagonal slepton mass matrix (in the same basis in which the charged leptons mass matrix is diagonal) and taking into account only the contribution of pure Wino of mass \tilde{m} one finds for $I_A \equiv I(y_A)$ the following form

$$I_A = \frac{g^2}{32\pi^2} \left\{ -\frac{1}{y_A} + \frac{y_A^2 - 1}{y_A^2} \ln(1 - y_A) - (A \rightarrow \mu) \right\} \quad (14)$$

where $y_A = 1 - (M_A/\tilde{m})^2$, M_A is the A -th charged slepton mass. It is easy to see that for $I_e \sim 10^{-3}$ and positive we need $M_e \approx 1.7M_{\mu,\tau}$ for the charged slepton masses.

Such slepton mass patterns can arise in models with inverted hierarchy [19], although the proposed mechanism may have other realizations. Being an operator of dimension five, the nontrivial neutrino mass matrix does not affect the evolution of the lepton Yukawa and slepton mass squared matrices. Hence, if there exists a basis at M_X in which both lepton Yukawa and slepton mass squared matrices are simultaneously diagonal (fermion-sfermion mass alignment [20] present e.g. in models with abelian $U(1)$ gauge symmetry [21]), this basis remains unchanged during the RG evolution to the electroweak scale and hence the slepton masses remain diagonal. Thus in this

case lepton flavor-violating decays such as $\mu \rightarrow e\gamma$ are not induced.

In Fig. 1 we illustrate the neutrino unification idea by displaying the supersymmetric evolution of solar and atmospheric neutrino mass splittings as a function of energy, from the high energy scale M_X where neutrino masses unify, down to the weak scale for $\tan\beta = 2$ and a starting neutrino mass at M_X of 1.2 eV. Above the supersymmetry breaking scale the neutrino mass-square differences are $\Delta m_{21}^2 = \frac{1}{3}\Delta m_{31}^2 = \frac{1}{2}\Delta m_{32}^2 \simeq -m^2 I_\tau$. After threshold corrections they change according to Eqs. (11,12,13). Depending on the chosen value of M_X this can fit large mixing angle solutions to the solar neutrino problem. For example for $M_X = 10^{13} \text{GeV}$ and $I_e \approx 2 \times 10^{-3}$, $r = \frac{I_\tau}{I_e} = -0.029$ we obtain quasi-degenerate neutrino masses $|m_i| = 0.9 \text{eV}$ split by $\Delta m_{21}^2 = -1.52 \times 10^{-5} \text{eV}^2$ and $\Delta m_{32}^2 = -3.13 \times 10^{-3} \text{eV}^2$. Neutrino mixing is bi-maximal $s_{12} \approx s_{23} \approx 0.707$ with $s_{13} \approx 0.0143$, nicely matching the LMA solution. In order to achieve the LOW solution one needs a smaller I_τ value. Barring a possible cancellation between the terms in the r.h.s of Eq. (13) this requires a low neutrino unification scale $M_X = 10^5 \text{GeV}$. In this case we find that quasi-degenerate masses $|m_i| = 1.1 \text{eV}$ for $r = \frac{I_\tau}{I_e} = -0.0103$ and $I_e \approx 1.3 \times 10^{-3}$ with $\Delta m_{21}^2 = -2.74 \times 10^{-7} \text{eV}^2$ and $\Delta m_{32}^2 = -3.16 \times 10^{-3} \text{eV}^2$. Mixing is nearly bi-maximal with $s_{13} \approx 0.0051$. Due to the observed flatness of the latest recoil electron energy spectrum, present solar neutrino data globally prefer large mixing angle MSW solutions over the small mixing solution (SMA) [4]. This feature fits well in our neutrino unification scheme, where the SMA solution can *not* be realized. From ref. [4] we find that in our scheme the LMA solution is present at the 97% C.L. while the LOW solution appears already at 90% C.L.

Conclusions.

We have proposed a simple theoretical ansatz where neutrino masses arise from a common dimension-5 operator at some ‘‘neutrino unification’’ scale M_X . This scale must be lower than that which characterizes the unification of gauge and Yukawa couplings. The ansatz naturally implies quasi-degenerate neutrinos at the electron-volt range, thus accessible to future β decay and hot dark matter searches. However neutrinoless double beta decay is naturally suppressed due to the intrinsic Majorana neutrino CP parities. We have shown that neutrino splittings can be induced by supersymmetric thresholds arising from non-universal soft breaking terms, without necessarily conflicting with limits on lepton flavour violating processes such as $\mu \rightarrow e\gamma$. A non-trivial consequence of the neutrino unification idea is that the solar and atmospheric data can only be accounted for in terms of large mixing angle-type MSW oscillations. Note also that the consistency of our scheme correlates the smallness of θ_{13} indicated by reactor data and, to a lesser extent, also by

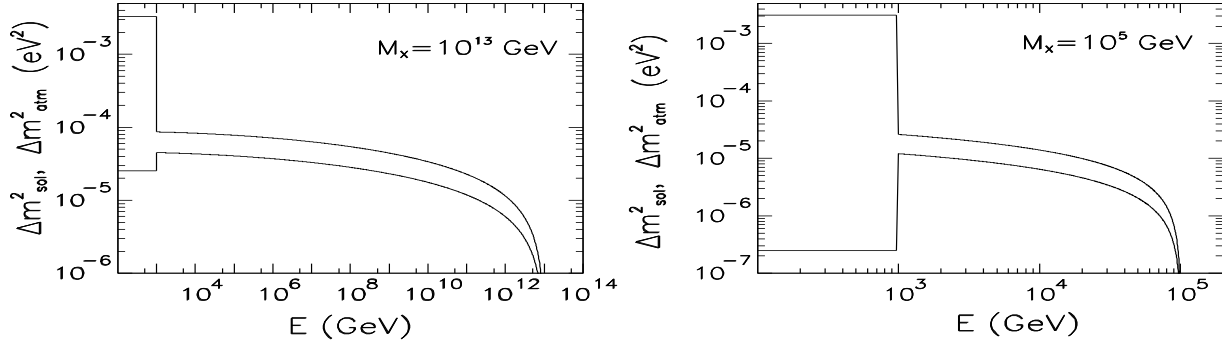


FIG. 1. Schematic evolution of solar and atmospheric splittings corresponding to LMA (left) and LOW solutions (right)

the atmospheric data [4], to the largeness of θ_{12} required by the solar neutrino data.

Acknowledgements: Work supported by Spanish DG-ICYT under grants PB98-0693 and SAB/1998-0136 (A.I.), by the European Commission TMR network HPRN-CT-2000-00148, by the European Science Foundation network grant N. 86 and by the Polish State Committee for Scientific Research grant KBN 2 P03B 060 18. P.H.Ch. thanks the CERN Theory group for hospitality during the completion of this paper.

-
- [1] Y. Suzuki, talk at XIX International Conference on Neutrino Physics and Astrophysics, Sudbury, Canada, <http://nu2000.sno.laurentian.ca>; T. Takeuchi, talk at the XXXth International Conference on High Energy Physics, ICHEP 2000 <http://www.ichep2000.rl.ac.uk>. Homestake Collaboration, B.T. Cleveland *et al.*, *Astrophys. J.* **496**, 505 (1998); R. Davis, *Prog. Part. Nucl. Phys.* **32**, 13 (1994); K. Lande, <http://nu2000.sno.laurentian.ca>; SAGE Collaboration, J.N. Abdurashitov *et al.*, *Phys. Rev.* **C60**, 055801 (1999); V. Gavrin, <http://nu2000.sno.laurentian.ca>. GALLEX Collaboration, W. Hampel *et al.*, *Phys. Lett.* **B447**, 127 (1999). E. Bellotti, <http://nu2000.sno.laurentian.ca>.
- [2] H. Sobel, <http://nu2000.sno.laurentian.ca>; T. Toshito, <http://www.ichep2000.rl.ac.uk>.
- [3] R. Becker-Szendy *et al.*, *Phys. Rev.* **D46**, 3720 (1992). H.S. Hirata *et al.*, *Phys. Lett.* **B280**, 146 (1992); Y. Fukuda *et al.*, *ibid* **B335**, 237 (1994). W.W.M. Allison *et al.*, *Phys. Lett.* **B449**, 137 (1999). A. Mann, <http://nu2000.sno.laurentian.ca/T.Mann/index.html> F.Ronga, <http://www.ichep2000.rl.ac.uk/Program.html>. See also hep-ex/0001058
- [4] M.C. Gonzalez-Garcia, M. Maltoni, C. Pena-Garay and J.W.F. Valle, *Phys. Rev. D* **63**, 033005 (2001) [hep-ph/0009350] and references therein.
- [5] H. Georgi, H. Quinn, and S. Weinberg, *Phys. Rev. Lett.* **33** 451 (1974).
- [6] S. Weinberg, Opening Address - Neutrinos '78, *In West Lafayette 1978, Proceedings, Neutrinos '78*.
- [7] J. Schechter and J. W. F. Valle, *Phys. Rev.* **D22**, 2227 (1980).
- [8] As shown in J. Schechter and J. W. F. Valle, *Phys. Rev.* **D23** (1981) 1666 Majorana phases are physical, though their effects are small due to the small neutrino masses.
- [9] For a recent discussion see S.M. Bilenki, C. Giunti and W. Grimus, *Prog. Part. Nucl. Phys.* **48**, 1 (1999).
- [10] Degenerate states can mix even if massless, see J. W. F. Valle, *Phys. Lett.* **B199**, 432 (1987) and *Prog. Part. Nucl. Phys.* **26** (1991) 91.
- [11] J. Schechter and J. W. F. Valle, *Phys. Rev.* **D24**, 1883 (1981) & **D25**, 283 (1982)
- [12] C. Wetterich, *Nucl. Phys.* **B187**, 343 (1981); G. M. Asatrian and A. N. Ioannisian, *Sov. J. Nucl. Phys.* **51**, 115 (1990); P. H. Chankowski and Z. Pluciennik, *Phys. Lett.* **B316**, 312 (1993); K. S. Babu, C. N. Leung and J. Pantaleone, *Phys. Lett.* **B319**, 191 (1993).
- [13] P. H. Chankowski, W. Królikowski and S. Pokorski, *Phys. Lett.* **B473** (2000) 109; J.A. Casas, J.R. Espinosa, A. Ibarra and I. Navarro, *Nucl. Phys.* **B573**, 652 (2000).
- [14] E. J. Chun, S. Pokorski, *Phys. Rev.* **D62**, 053001 (2000).
- [15] J. Ellis and S. Lola, *Phys. Lett.* **B458**, 310 (1999); J.A. Casas, J.R. Espinosa, A. Ibarra, I. Navarro *Nucl. Phys.* **B569** 82 (2000); N. Haba, Y. Matsui, N. Okamura, hep-ph/0005075; K. R. Balaji *et al* *Phys. Rev. Lett.* **84** (2000) 5034; E. Ma, UCRHEP-T260, talk at NANP99, Dubna
- [16] D.E. Groom *et al.*, *The European Phys. J.* **C15**, 1 (2000)
- [17] M. Apollonio *et al.*, *Phys.Lett.* **B466** (1999) 415, hep-ex/9907037; F. Boehm *et al.*, hep-ex/9912050
- [18] L. Wolfenstein, *Phys.Lett.* **B107** (1981) 77; J. W. F. Valle, *Phys. Rev.* **D27**, 1672 (1983); J. W. F. Valle and M. Singer, *Phys. Rev.* **D28**, 540 (1983).
- [19] E. Dudas, S. Pokorski and C.A. Savoy, *Phys. Lett.* **B369**, 255 (1996); E. Dudas, Ch. Grojean, S. Pokorski and C. Savoy, *Nucl. Phys.* **B481** (1996), 85; A.G. Cohen, D.B. Kaplan and A.E. Nelson, *Phys. Lett.* **B388** (1996), 588. Ph. Brax and C.A. Savoy, *JHEP0007:048* (2000).
- [20] Y. Nir and N. Seiberg, *Phys. Lett.* **B309** (1993), 337; P. Binetruy *et al* *Nucl. Phys.* **B496** (1997) 3.
- [21] L. Ibanez and G. Ross *Phys.Lett.* **B332** (1994) 100; P. Binetruy and P. Ramond *Phys.Lett.* **B350** (1995) 49; Y. Grossman and Y. Nir *Nucl.Phys.* **B448** (1995) 30.